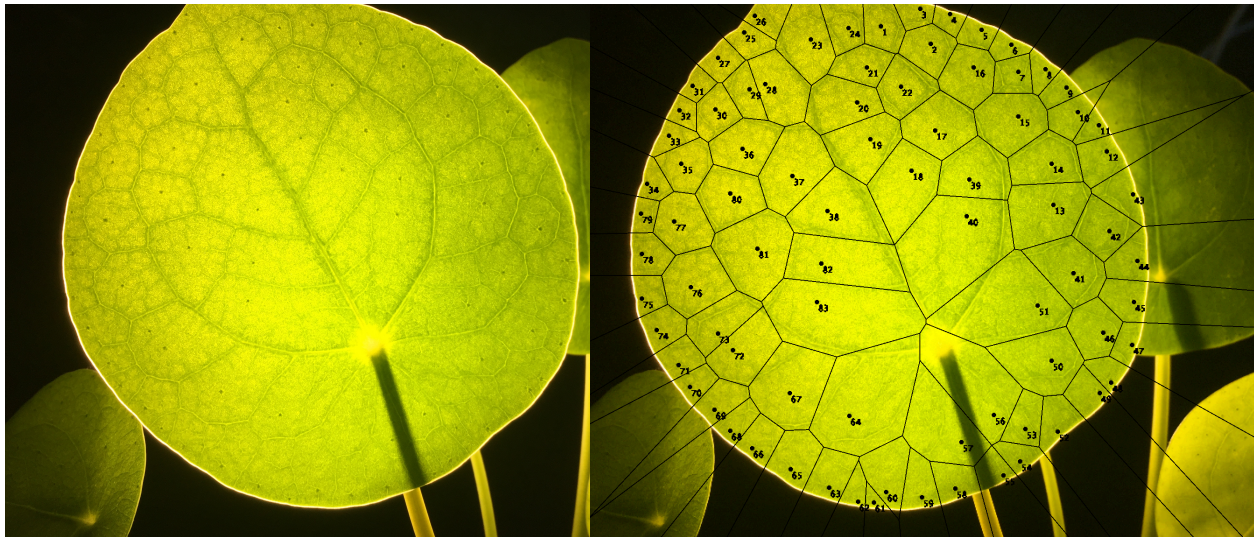


Sunlight shone through the leaves, and I saw the veins and Pilea's secret illuminated.



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A few years ago my sister brought me a dying houseplant, trusting that I would revive it. At the time I had just started working with Professor [Saket Navlakha](#) at the Cold Spring Harbor Laboratory, researching how veins form in leaves. He had a hypothesis that perhaps the very smallest veins, the reticulate venation, follow a pattern called a Voronoi diagram. As we shall later see, in this plant, the largest rather than smallest, veins, follow this pattern.

What is a Voronoi diagram? Imagine that you are dropped out of the sky into the calm ocean, in the midst of several tiny islands. Each of the islands have motorboats which travel at the same speed. Which island would you like the rescuers to come from? The closest island of course! Since people falling out of the sky is a common occurrence, these tiny islands have drawn up a map, cutting up the ocean into regions in which each island or seed point is closest. This map is a Voronoi diagram. Voronoi diagrams turn up often in nature but always missing the seed points (islands in our analogy). They are also useful tools in mathematics and computing.

While I was busy reading papers on how veins form, and designing and programming computer simulations, I also spent a lot of time thinking about how to test the hypotheses, how to collect vein data, and which plants' leaves to use. All this time, I was also taking care of my sister's houseplant. As sunlight shone through the leaves, I saw the veins and Pilea's secret illuminated.

In the photo we see small dots, surrounded by regions divided by the major veins. This beautiful pattern is in fact a Voronoi diagram! To our knowledge this is the first example of a Voronoi diagram in nature which comes together with visible seed points.

I quickly shared this discovery with Saket and we started asking completely new questions:



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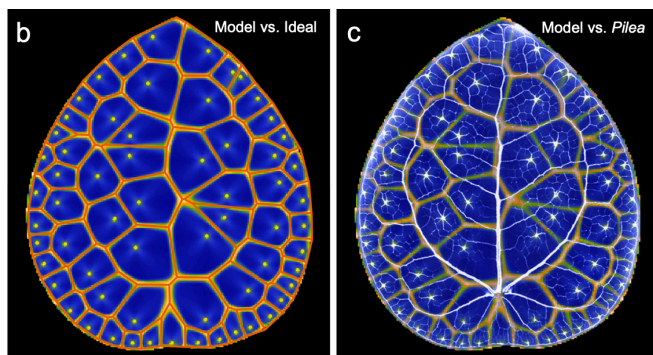
First of all what are these dots? How do we know it really is a Voronoi diagram? And why is it forming this pattern? Is it true for other plants? Here are answers in short, which took a long time to uncover. For more details, see the paper:

I. The dots are hydathodes, within each of which lies a small druse crystal. They play a role in water regulation.

II. Saket designed four statistical tests based on certain mathematical properties of Voronoi diagrams:

1. The line connecting two seed points is bisected by the Voronoi cell
2. It is bisected perpendicularly
3. How does the tessellation generated veins differ from the true Voronoi diagram generated by the hydathodes? (Using the Jaccard index)
4. Are the hydathodes really the seed points? How do they compare with centroids or midpoints of the polygons?

III. Why is it forming this pattern? There is evidence to suggest that the hydathodes act as sources of the hormone auxin. The leading theory for the formation of veins in leaves is the canalization hypothesis. Veins form as auxin is transported through the growing leaf, in particular certain proteins, PIN, become polarized as they facilitate the transport. I designed computer models for this mechanism very early on in the project, and later on Professor Przemyslaw Prusinkiewicz designed more sophisticated simulations based on the canalization hypothesis. His simulations were



able to generate similar patterns to those observed in *peperomioides*'s leaves. This model gives a glimpse into the world of biologically distributed algorithms, how organisms compute things like Voronoi diagrams with their bodies.

IV. Is it true for other plants? Our results support that the canalization hypothesis can generate a wide variety of patterns including Voronoi patterns. More specifically, do we find Voronoi diagrams in other plants' venation? This is a great question, and unclear at the moment. We were unable to find any others which were so clearly Voronoi diagrams. That's one thing that makes *Pilea peperomioides* so special. Even the closest relatives of *Pilea peperomioides* actually have very different leaf structure from it. Nevertheless, I believe one close relative, *Pilea pumila*, might show something similar.

We have just glimpsed one of the secrets of nature. Thanks for joining me.

I presented the initial research in the April 2021 PFF Symposium at the CSHL. Please see the [paper](#) for more details.

I have always been fascinated by patterns in nature. Today, I continue to research patterns in nature as a PhD student in mathematics.